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AN ESTIMATE OF THE EFFECTS OF A LAMINAR EQUILIBRIUM BOUNDARY LAYER ON THE DOWNSTREAM WAKE

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By Aerodynamics Department

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I INTRODUCTION

In order to conduct a detailed investigation of the communication and detection problems which result from ionization of the flow field of reentry vehicles moving through the atmosphere, the effect of the plasma sheath upon the electromagnetic waves must be analyzed. This requires the detailed knowledge of the structure of the flow about the body. Correspondingly, a similar significance is attached to the problem of the determination of the structure of the wake behind the body since the plasma sheath extending downstream has the property of reflecting or absorbing the same electromagnetic waves. As a result of the high velocity during the passage of the body through the atmosphere, the composition of the flow field between the shock wave and body as well as the wake is found to consist of dissociated and ionized air. In this report the problem considered is that of the contribution of the laminar boundary layer to the structure of the flow field and whether its effect may be significant for inclusion in the detailed analysis for the chemical species in the wake.

A reasonably simplified model of the flow field is all that is required to develop an understanding of the physical situation and also permit rapid

computations to be carried out. The dissociated and ionized air in the flow is considered separately as the boundary layer close to the body and an inviscid flow filling the balance of the field out to the shock. This simplified model contains the essentials for the evaluation of the boundary layer about the body for the purpose of this study. From this investigation, the problem of whether or not it is necessary to include the effects of a dissociated laminar boundary layer in the analysis of the wake can be ascertained.

Equilibrium conditions were assumed behind the shock and in the boundary layer so that the solutions for the local flow properties, in particular electron densities, are readily obtainable from tabulated data. Solutions for the boundary layer over a cooled wall with a pressure gradient were taken from Reference 1, so that the distributions for the dissociated, ionized equilibrium flow within the layer could readily be obtained. For conditions where evaluation of the equilibrium boundary layer indicates a possibility of contributing significantly to departures from the electron density distribution in the inviscid field (which would not be accounted for in an inviscid field analysis) a more exact solution would then be required.

The investigation of the nonequilibrium flow can then be carried out utilizing the solutions from the equilibrium analysis. The nonequilibrium flow solutions are obtainable through the introduction of reasonable approximations for the important reactions in order to account for the rate processes which yield different chemical species distributions and electron densities.

The problem of the determination of the effect of a dissociated laminar boundary layer upon the structure of the wake downstream of the reentering body could then be carried out. The results from this analysis for the structure of the boundary layer and external flow field at the rear of the body supply the initial conditions of the flow in the analysis of the wake.

II ANALYSIS

In order to study the effects of the dissociated boundary layer a reasonable approximation to the inviscid flow field was required. In another report, the results of a high speed computer program for the more accurate inviscid field about the body will be presented. The flow conditions used here in the inviscid approximation are the shock shape and equilibrium conditions are assumed immediately behind the shock and surface conditions at the body (outer edge of boundary layer) for the several altitudes specified corresponding to the trajectory conditions supplied to GASL in Reference 2. The coordinate system used and the body configuration are shown in Figure 1.

The shock shape and standoff distance from equations (41a, b, c) of Reference 3 are given (in coordinate system with origin at standoff distance away from nose on the center line) by

$$x = ay^2 + by^4 \tag{1a}$$

$$\Delta = \frac{2}{3} \frac{R_N}{\rho_2/\rho_1 - 1} \tag{1b}$$

where ρ_2/ρ_1 is the density ratio across the normal shock. For the hemispherical nose, $b = 0$ and

$$a = \frac{1}{2} (R + \Delta)^{-1} \tag{1c}$$

The pressure distribution over the surface of the body was determined from a modified Newtonian relation to 70° from the stagnation point. From the 70° location to the shoulder, 90°, the pressure distribution will closely

correspond to that given by a constant γ, Prandtl-Meyer expansion . Over the short cylindrical section the pressure is maintained constant and again a Prandtl-Meyer expansion down to the rearward facing frustum was used. Compression corresponding to a flow deflection parallel to the rear cylinder provides the pressure at the end of the body. With the shock shape given by equations (1) and the pressure distribution on the body determined, all other flow properties can readily be found along these boundaries for the equilibrium flow.

The analysis of the boundary layer was then carried out with the conditions previously determined for the inviscid field along the contour of the body as the outer edge properties of the boundary layer. In this report the important flow parameter resulting from the analysis is the mass flow through the boundary layer. The ratio of the boundary layer mass flow to other mass flows discussed below forms the basis for considerations of the effects of boundary layer dissociation.

The total mass flow rate through the boundary layer, at any station x, is given by

$$m_{b.L.} = 2\pi r(x) \int_0^{\delta(x)} \rho u dy \tag{2}$$

Introduction of the transformed coordinates, $\tilde{s}$ defined by

$$\tilde{s} = \int_0^x \rho_e u_e \mu_e r^2 \, dx \tag{3a}$$

and the nondimensional normal, η, given by

$$\eta = \frac{\rho_e u_e}{\sqrt{2\tilde{s}}} \int_0^y \frac{\rho}{\rho_e} r dy \qquad (3b)$$

reduces the equation(2)to

$$m_{b.L.} = 2\pi \sqrt{2\tilde{s}} \int_0^{\eta_\delta} \frac{u}{u_e} d\eta \qquad (4)$$

The equation for $\tilde{s}$ contains terms which are functions only of boundary layer outer edge conditions. The integral containing the velocity ratio, in the transformed coordinate η, can now be evaluated with many solutions available in the literature for velocity profiles corresponding to the conditions of interest.

Interest in this report was focused upon the use of metallic wall materials which have melting points of up to (approximately) 1200° K. Since the reentering body experiences a transient heating until it melts at some point along the trajectory, the ratio of wall temperatures to outer edge of boundary layer temperatures will approach a maximum of about 0, 35. During the time of reentry to destruction the wall temperature ratios remain less than this value so that it is possible to consider the wall condition as a relatively cold wall. Thus the velocity profiles introduced to calculate the integral of equation 4 must correspond to the cold wall condition.

The boundary layer thickness is given by

$$\delta = \frac{\sqrt{2\tilde{s}}}{\rho_e u_e r_e} \int_0^{\eta_\delta} \frac{\rho_e}{\rho} \, d\eta \tag{5a}$$

and the displacement thickness is obtained from

$$\delta^* = \frac{\sqrt{2\tilde{s}}}{\rho_e u_e r_e} \int_0^{\eta_\delta} \left(\frac{\rho_e}{\rho} - \frac{u}{u_e} \right) d\eta \tag{5b}$$

Since a simplified method of solution is to be used in the numerical computations where assumed velocity profiles are introduced, the results of computation of the above equations will tend to be more inaccurate at high altitudes. The exact solutions for boundary layer distributions over a spherical nose at hypersonic velocities remains a difficult problem. For the purpose of the present investigation where mass ratios are of primary interest, the use of assumed velocity profile solutions will be quantitatively useful. For discussion of the high altitude interaction problem see Reference 4.

The enthalpy ratio is substituted for density ratio, $\frac{\rho_e}{\rho} = \frac{h}{h_e}$, given by

$$\frac{\rho_e}{\rho} = \frac{h}{h_e} = \frac{h_{se}}{h_e} \left[\frac{h_w}{h_{se}} + \left(1 - \frac{h_w}{h_{se}} \right) \frac{u}{u_e} - \left(\frac{u}{u_e} \right)^2 \right] + \left(\frac{u}{u_e} \right)^2 \tag{6}$$

Another mass flow within the boundary layer which will be considered here is that portion of boundary layer flow containing all values of electron densities up to the specified level N_1. The equation is the same as equation (4) where the upper limit of the integral is replaced by η_{N_1}. The partial mass flows obtained represent the portions of the boundary layer associated with reduced (for the cool wall) electron densities.

The reference mass flow of interest is the free stream mass flow contained within a stream tube which intersects the shock and results in specified levels of electron density immediately behind the shock and it is designated by $(m_\infty)_N$. The radius of the stream tube which encloses the flow $\rho_\infty u_\infty$ which intersects the shock given by equations (1) is determined for the equilibrium case by specifying the lower electron density levels which are to be considered. The normal velocity component of the free stream at the point on the shock where the electron density level occurs at a given altitude can be determined from the charts of Reference 5. The slope at this intersection point of the shock and stream tube is then determined; this follows from the slope equation (in the shock coordinates)

$$\frac{dy}{dx} = \frac{1}{2ay} \tag{7}$$

from which the point x, y on the shock is found. The value of y is the desired stream tube radius for the entering mass flow across the shock which contains electron densities behind the shock from the maximum value at the stagnation point down to the specified value N indicated for the several altitudes.

III RESULTS

The use of a constant γ approximation permits rapid computation of the pressure distribution around the body; it was used to obtain the outer edge boundary layer conditions. Pressure distributions for the unseparated laminar boundary layer at altitudes of 150,000, 200,000 and 250,000 feet were obtained as described in the analysis. The electron density distributions about the body at these altitudes were determined as a function of temperature and pressure using the charts in Reference 6.

The mass flow entering across the assumed shock shape with the specified limiting electron densities of $N = 10^8$, 10^9, 10^{10} electrons per cc were obtained as previously described. The ratios of boundary layer mass flow, from equation (4), to the entering mass across the shock were computed for the shoulder, $x = 3.96$ in. In Figure 2, the mass ratios are plotted against the altitudes for the several values of the electron density, N. The mass flow ratios for altitudes up to 225,000 ft. are less than 10% and are increasing rapidly but remain less than 20% at 250,000 ft. Since the boundary layer mass flow represents a sufficiently small fraction of the entering mass flow across the shock (which is constituted of sufficiently high electron densities to be of interest in the electromagnetic wave problem) then the inviscid flow analysis presents the electron field around the body with sufficient accuracy for study of the wake. Downstream of the body, as the pressure is reduced to ambient, the boundary layer mass flow in the starting region of the wake can be considered (approximately) as an annulus contracting in size and still further downstream contracts to a small cylindrical core.

The balance of the wake region consists of flow from the inviscid field around the body. It appears then that the electron density distributions associated with the boundary layer need not be considered in the far downstream wake.

Within the boundary layer in the relatively cool region close to the wall (which is absorbing heat from the air) the electron densities fall off sharply if equilibrium conditions prevail. In Figure 3, for the 250,000 ft. case, the electron distributions in the boundary layer at the shoulder are shown against the mass ratio of partial boundary layer mass flow to mass flow through the shock resulting in electron densities of 10^8, 10^9 and 10^{10} electrons per cc. The fractional boundary layer mass is computed for different heights normal to the body surface from equation (4) with the substitution of η_N for the upper limit. Less than 3% of the mass ratio has electron densities approaching 10^{10} electrons per cc, while the balance of the boundary layer contains essentially the same electron density as the inviscid flow. This indicates that for purposes of this investigation, the bulk of the boundary layer flow which is at higher electron densities may be assigned to the inviscid flow regime for further consideration in the wake analysis. Thus the initial conditions of the wake analysis from the electron distribution contained in the flow field at the rear of the body does not require that the boundary layer be considered as a distinct and separate portion of that field.

The results of a similar analysis for the nonequilibrium boundary

layer would not yield any significant differences with respect to the conclusions concerning the formulation of the wake problem. Locally, of course, composition of boundary layer air would be altered and electron densities changed; however, mass flow ratios would not be appreciably different. Therefore, the previous comments can be considered applicable to the nonequilibrium case insofar as consideration of boundary layer flow in the wake is concerned.

IV CONCLUSIONS

The electron distribution in the flow field at the rear of the body can be satisfactorily represented without taking into account the presence of the boundary layer. For the 2 1/2-inch-nose-radius body considered herein, the contribution of the equilibrium laminar boundary layer to the dissociated ionized flow may be neglected for further analysis of the wake.

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LIST OF SYMBOLS

$\frac{h}{h_e}$	enthalpy ratio (local to outer edge of boundary layer)
h_{se}	stagnation enthalpy at edge of boundary layer
h_w	wall enthalpy
$m_{b.L.}$	mass flow in the boundary layer
$(m_\infty)_N$	mass flow entering shock based on conditions at ∞ with electron density, N, behind shock at intersection with stream tube
N_1	electron density within boundary layer at η distance from wall
R_N	nose radius
$r(x)$	radius of sections of body normal to centerline
$\tilde{s}$	transformed coordinate given by equation (3a)
$\frac{u}{u_e}$	ratio of local velocity in boundary layer to velocity at outer edge
x, y	shock coordinates (origin at stand-off distance from body), also body surface coordinates
$\delta(x)$	boundary layer thickness
Δ	stand-off distance of shock
η	transformed coordinate defined by equation (3b)
η_δ	value of η at boundary layer thickness δ
η_{N_1}	value of η corresponding to height in boundary layer where electron density N_1 occurs
$\frac{\rho_2}{\rho_1}$	density ratio across normal shock

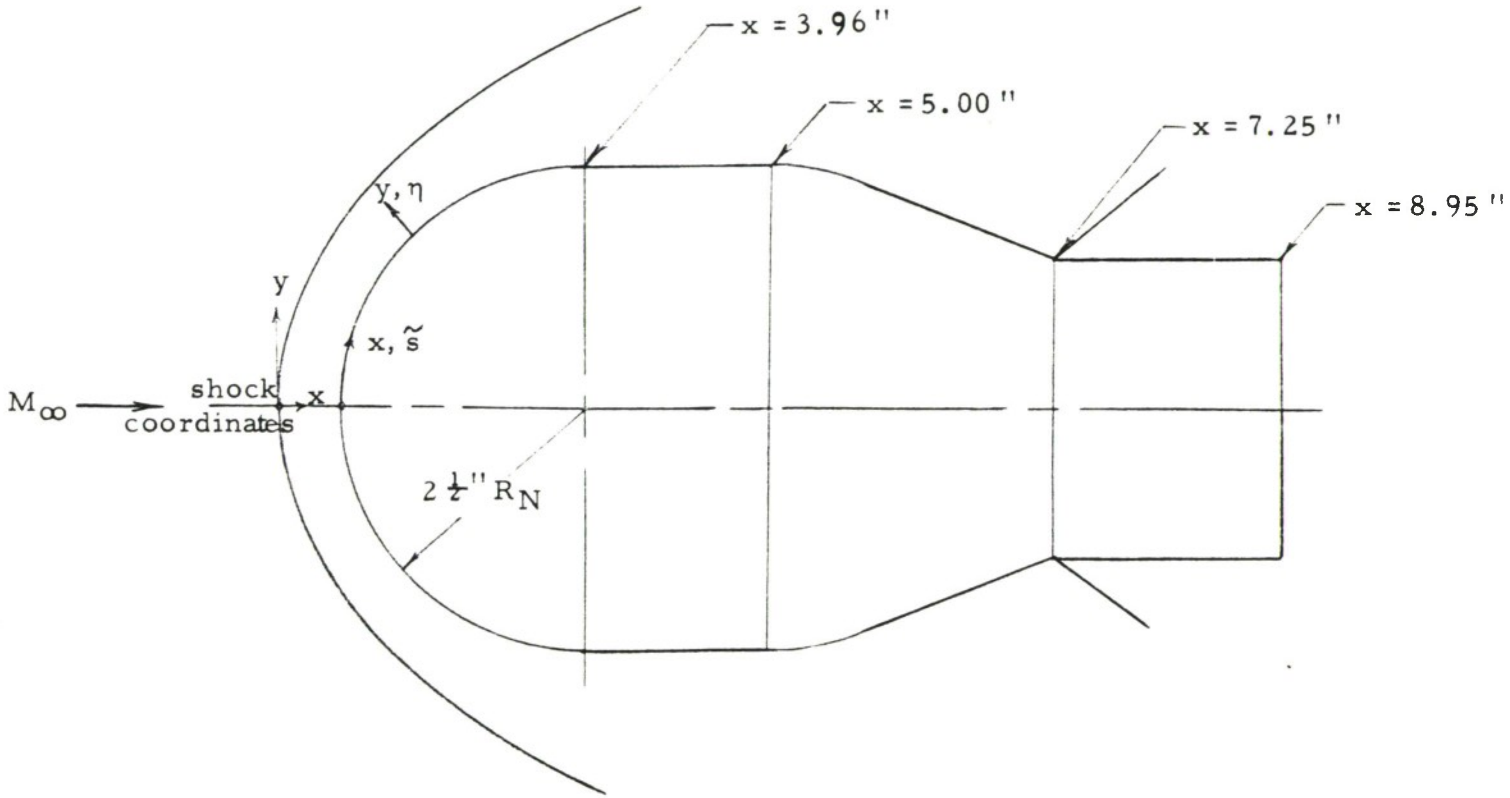
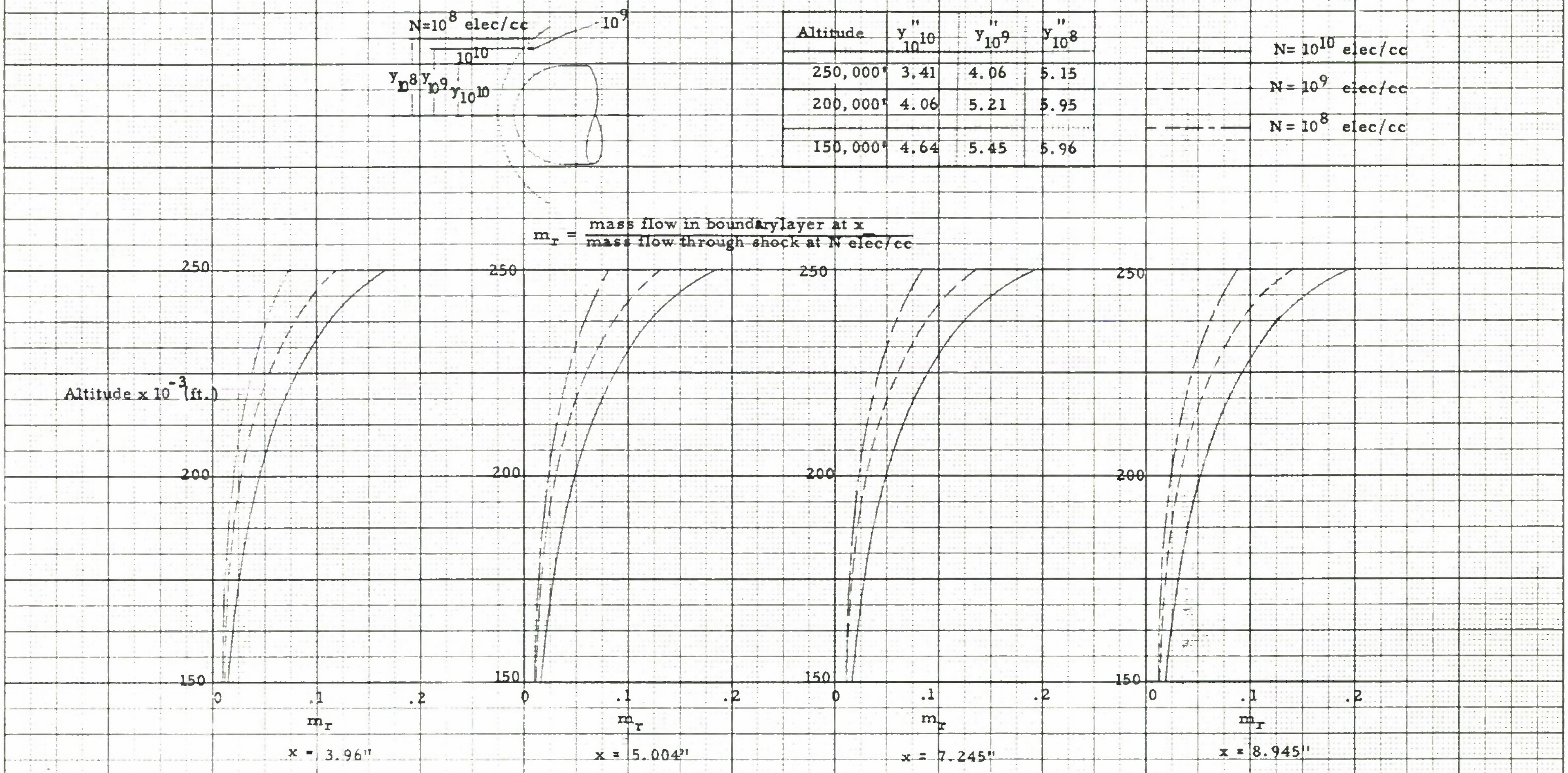


Figure 1. Reentry Body Configuration



Altitude	$y''_{10^{10}}$	y''_{10^9}	y''_{10^8}
250,000'	3.41	4.06	5.15
200,000'	4.06	5.21	5.95
150,000'	4.64	5.45	5.96

Figure 2. Mass Ratio of Boundary Layer to Shock Flow for Several Electron Densities

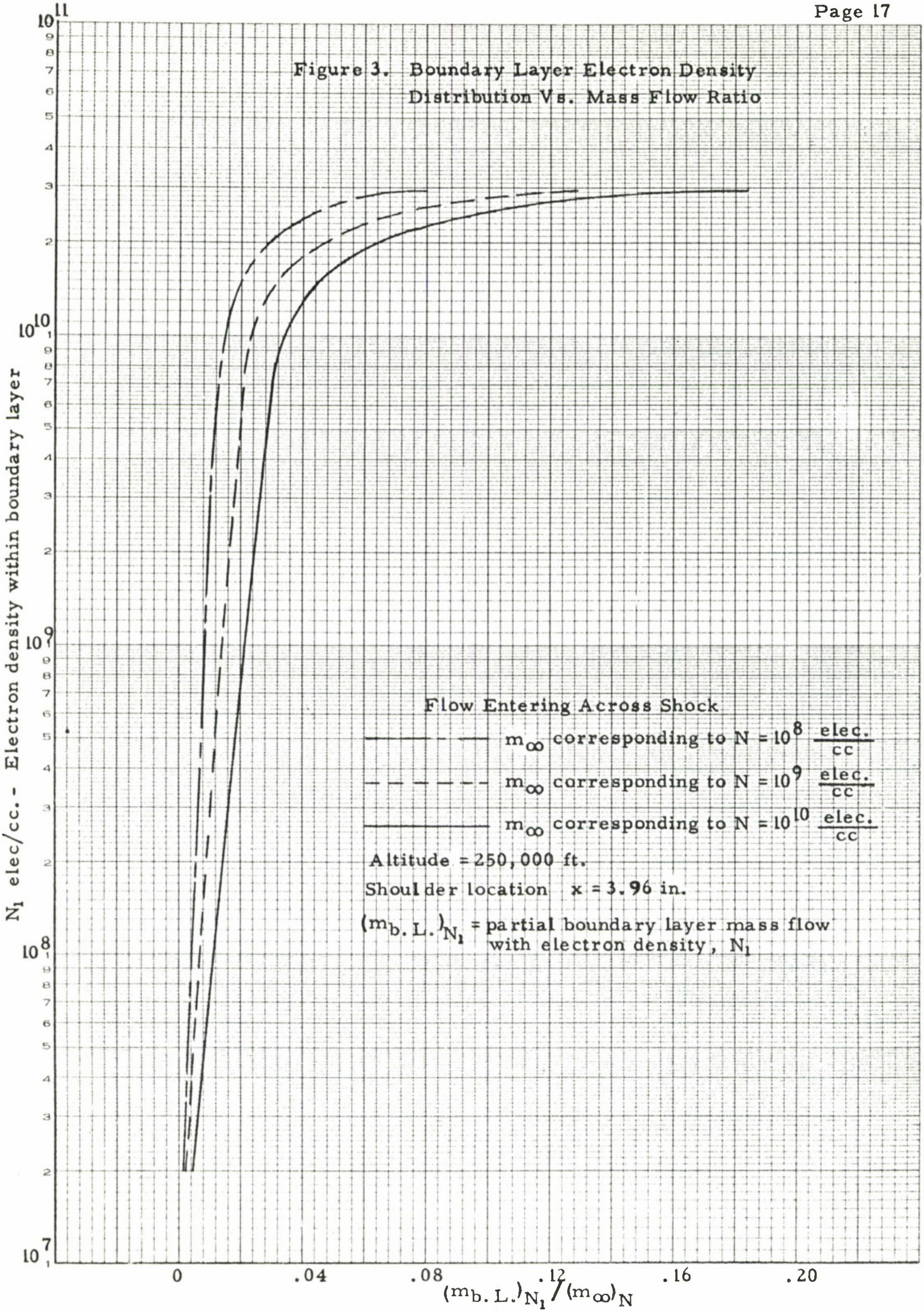


Figure 3. Boundary Layer Electron Density
Distribution Vs. Mass Flow Ratio
Flow Entering Across Shock
m_∞ corresponding to $N = 10^8$ $\frac{elec.}{cc}$
m_∞ corresponding to $N = 10^9$ $\frac{elec.}{cc}$
m_∞ corresponding to $N = 10^{10}$ $\frac{elec.}{cc}$
Altitude = 250,000 ft.
Shoulder location x = 3.96 in.
$(m_{b.L.})_{N_1}$ = partial boundary layer mass flow with electron density, N_1
N_1 elec/cc. - Electron density within boundary layer
10^{11}
10^{10}
10^9
10^8
10^7
0
.04
.08
.12
.16
.20
$(m_{b.L.})_{N_1}/(m_\infty)_N$